Chapter #2

Coulomb's law, Coulomb's Force

$$F = \frac{K Q_1 Q_2}{R^2}, \quad K = \frac{1}{4\pi\epsilon_0} = 9\times10^9 \ Nm^2/C^2$$

or $F = \dfrac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$

$\epsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$
$=$

$\downarrow$

$(C^2/Nm)/m$

$\downarrow$

F/m

→ If $Q_1 = Q_2 = 1C$
and $R = 1m$
then $F = 9\times10^9 N$

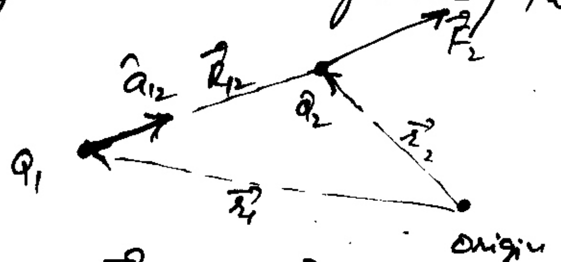
$\Rightarrow \epsilon_0 = 8.85\times10^{-12}\ F/m$

or $\epsilon_0 = \dfrac{1}{36\pi} 10^{-9}\ F/m$

Vector Form of Coulomb's Law

→ Coulomb's force acts along the line joining the two charges.

→ Repulsive (like charges)
Attractive (unlike charges)



→ let $Q_1 \rightarrow \vec{r}_1$
$Q_2 \rightarrow \vec{r}_2$

then $\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$

$$\Rightarrow \vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{R}_{12}|^2}\hat{a}_{12}, \quad \Rightarrow \hat{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$\vec{F}_2$ = Force on Q_2 by Q_1

Where $\hat{a}_{12}$ is a unit vector in the direction of $\vec{R}_{12}$.

$\Rightarrow \vec{F}_1$ = Force on Q_1 by Q_2 $= \dfrac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{R}_{21}|^2}\hat{a}_{21}$

$$\vec{F}_1 = -\left(\frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{R}_{12}|^2}\hat{a}_{12}\right)$$

$\Rightarrow \vec{F}_1 = -\vec{F}_2$

→ Coulomb's law is linear

i.e. $Q_1 \rightarrow nQ_1$
$\Rightarrow \vec{F}_2 \rightarrow n\vec{F}_2$

Also $Q = Q_1 \Rightarrow \vec{F}_1$
$Q_t = Q_1 + Q_2 + \dots + Q_n \rightarrow \vec{F}_t = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_N$

Electric Field Intensity

→ Force Field

→ Test charge Q_t feels a force everywhere on it due to Q_1

$$\vec{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 |\vec{R}_{1t}|^2}\hat{a}_{1t}$$

$$\Rightarrow \frac{\vec{F}_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 |\vec{R}_{1t}|^2}\hat{a}_{1t} = \vec{E}$$

$$\Rightarrow \vec{E} = \frac{\vec{F}_t}{Q_t} = \text{Electric Field Intensity}$$

(N/C or V/m) → $E = \frac{V}{d} = V/m$

→ $\vec{E}$ is function of Q_1 ~~only~~ and directed line segment from Q_1 to the position of Q_t.

→ $$\boxed{\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\hat{a}_{1t}}$$ → Defining expression for $\vec{E}$ (due to a single point charge)

Generalizing $$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\hat{a}_R}$$

→ Q_1 located at the centre of the spherical coordinate system, then

$$\vec{E} = \frac{Q_1}{4\pi\epsilon_0 r^2}\hat{a}_r, \quad R \to r$$

(Single radial component) & $\hat{a}_R \to \hat{a}_r$

In rectangular coordinate system

$$\vec{R} = \vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$\hat{a}_R = \hat{a}_r = \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{a}_x + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{a}_y + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{a}_z$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 (x^2+y^2+z^2)}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{a}_x + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{a}_y + \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

→ When Q is not located at the centre of our coordinate system, spherical symmetry is lost.

$$Q \to \vec{r}\,' = x'\hat{a}_x + y'\hat{a}_y + z'\hat{a}_z$$

the field at a general point

$$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \quad \text{is given by}$$

$$\vec{E} = \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|^2} \cdot \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|} = \frac{Q(\vec{r} - \vec{r}\,')}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|^3}$$

$$= \frac{Q\left[(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z\right]}{4\pi\epsilon_0 \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^3}$$

$$\vec{E}(\vec{r}) = \frac{Q\left[(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z\right]}{4\pi\epsilon_0 \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

When $x' = y' = z' = 0$, then

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 \left|\sqrt{x^2+y^2+z^2}\right|^2} \cdot \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}$$

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 (x^2+y^2+z^2)}\left[\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{a}_x + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{a}_y + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{a}_z\right]$$

→ For n point charges

$$\vec{E}(\vec{r}) = \frac{Q_1}{4\pi\epsilon_0 |\vec{r} - \vec{r}_1|^2}\hat{a}_1 + \frac{Q_2}{4\pi\epsilon_0 |\vec{r} - \vec{r}_2|}\hat{a}_2 + \dots + \frac{Q_n}{4\pi\epsilon_0 |\vec{r} - \vec{r}_n|^2}\hat{a}_n$$

$$= \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\vec{r} - \vec{r}_m|^2}\hat{a}_m$$

Field Due To a Continuous Volume Charge Distribution

$$\Delta Q = \rho_v \, \Delta V$$

→ Volume charge density $\rho_v = \lim_{\Delta v \to 0} \dfrac{\Delta Q}{\Delta v}$

⇒ Total charge within a finite volum

$$Q = \int_{vol} \rho_v \, dv = \iiint \rho_v \, dv \quad \text{(Triple integration)}$$

→ Incremental contribution to $\vec{E}$ at $\vec{r}$ due to an incremental charge ΔQ at $\vec{r}\,'$ is

$$\Delta \vec{E}(\vec{r}) = \frac{\Delta Q}{4\pi\epsilon_0 |\vec{r}-\vec{r}\,'|^2} \cdot \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|} = \frac{\rho_v \Delta v}{4\pi\epsilon_0 |\vec{r}-\vec{r}\,'|^2} \cdot \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

Let $\Delta v \to 0$, then the total $\vec{E}$ due to infinite elements

$$\vec{E}(\vec{r}) = \int_{vol} \frac{\rho_v(\vec{r}\,')\, dv'}{4\pi\epsilon_0 |\vec{r}-\vec{r}\,'|^2} \cdot \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

$\vec{r}$ → point where $\vec{E}$ is to be determined

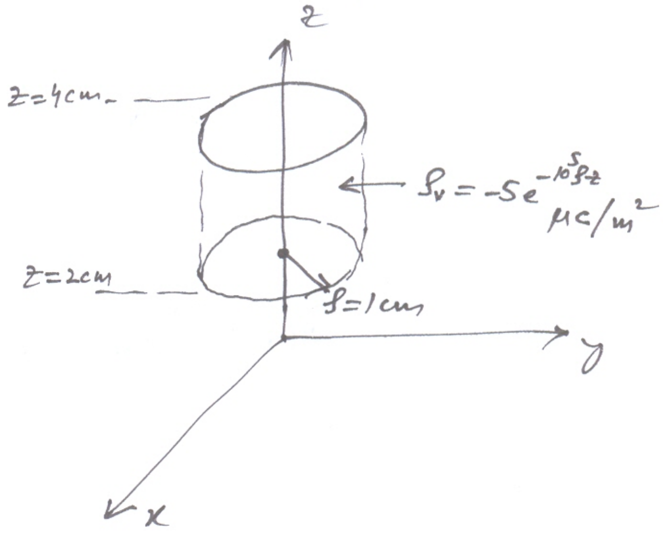
$\vec{r}\,'$ → point where $\underbrace{\rho_v(\vec{r}\,')\, dv'}_{dQ}$ is located

$|\vec{r}-\vec{r}\,'|$ → scalar distance b/w source point & field point

$\dfrac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$ → unit vector located from source point to field point.

Example 2.3 :

$Q = ?$



$$Q = \int_{vol} \rho_v \, dv$$

$$= \int_{0.02}^{0.04}\int_{0}^{2\pi}\int_{0}^{0.01} -5\times10^{-6}\, e^{-10^5 \rho z} \, \rho \, d\rho \, d\phi \, dz$$

$$\cancel{= \int_{0.02}^{0.04}\int_{0}^{0.01} -5\times10^{-6} \frac{e^{-10^5 \rho z}}{-10^5 \rho} \cdot \rho \, d\rho \, d\phi \, dz \Big|_0^{2\pi}}$$

$$\cancel{= \int_{0.02}^{0.04}\int_{0}^{0.01} -5\times10^{-11} e}$$

$$Q = \int_{0.02}^{0.04}\int_{0}^{2\pi}\int_{0}^{0.01} -5\times10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

→ First integrating w.r.t ϕ

$$Q = \int_{0.02}^{0.04}\int_{0}^{0.01} -5\times10^{-6} e^{-10^5 \rho z} \phi \, \rho \, d\rho \, dz \Big|_0^{2\pi}$$

$$Q = \int_{0.02}^{0.04}\int_{0}^{0.01} -5\times10^{-6} e^{-10^5 \rho z} (2\pi) \rho \, d\rho \, dz$$

$$Q = \int_{0.02}^{0.04}\int_{0}^{0.01} -10^{-5}\pi e^{-10^5 \rho z} \rho \, d\rho \, dz$$

→ Now integrating w.r.t z

$$Q = \int_{0}^{0.01} -10^{-5}\pi \cdot \frac{e^{-10^5 \rho z}}{-10^5 \cancel{\rho}} \cancel{\rho} \, d\rho \Big|_{0.02}^{0.04}$$

$$= \int_{0}^{0.01} +10^{-10}\pi \, e^{-10^5 \rho z} \Big|_{0.02}^{0.04} d\rho$$

$$Q = \int_{0}^{0.01} 10^{-10}\pi \left(e^{-4000\rho} - e^{-2000\rho} \right) d\rho$$

→ Now integrating w.r.t ρ

$$Q = 10^{-10}\pi \left(\frac{e^{-4000\rho}}{-4000} - \frac{e^{-2000\rho}}{-2000} \right) \Big|_0^{0.01}$$

$$= 10^{-10}\pi \left(\frac{e^{-2000\rho}}{2000} - \frac{e^{-4000\rho}}{4000} \right) \Big|_0^{0.01}$$

$$= 10^{-10}\pi \left\{ \left(\frac{e^{-20}}{2000} - \frac{e^{-40}}{4000} \right)^{0} - \left(\frac{1}{2000} - \frac{1}{4000} \right) \right\}$$

$$= 10^{-10}\pi \left(\frac{1}{2000} - \frac{1}{4000} \right)$$

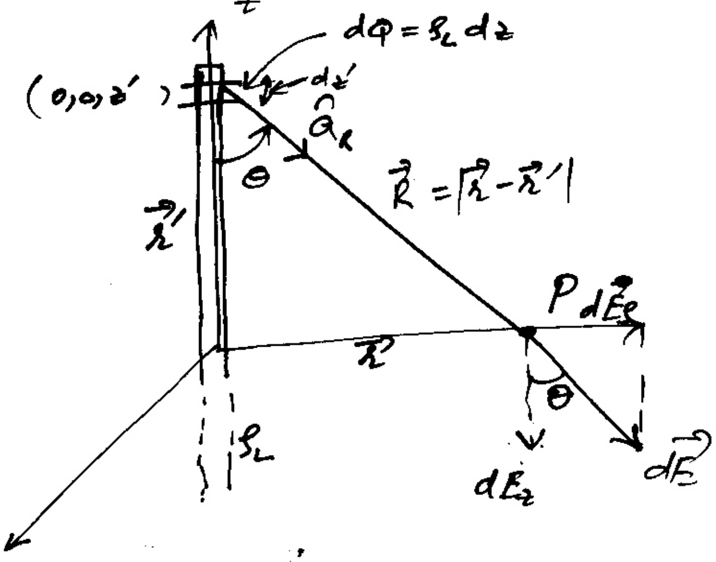
$$\boxed{Q = 0.0785 \, pC}$$

Field Of A Line Charge

→ Sharp beam in cathode-ray tube or a charged conductor of very small radius

Line charge density $= \rho_L = \dfrac{Q}{L}$ (C/m)

$dQ = \rho_L \, dz'$



→ No component in ϕ direction present, so $\vec{E}_\phi = 0$

→ z-components present, but cancelled out.

→ Only ρ-components add up, hence $\vec{E} = E_\rho \hat{a}_\rho$

Now $$d\vec{E} = \frac{dQ}{4\pi\epsilon_0} \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} = \frac{\rho_L \, dz'}{4\pi\epsilon_0} \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

Now $\vec{r} = y\hat{a}_y = \rho\hat{a}_\rho$

$\vec{r}' = z'\hat{a}_z$ $\Rightarrow \vec{r} - \vec{r}' = \rho\hat{a}_\rho - z'\hat{a}_z$

$|\vec{r} - \vec{r}'| = \sqrt{\rho^2 + z'^2}$

$$\Rightarrow d\vec{E} = \frac{\rho_L \, dz'}{4\pi\epsilon_0} \cdot \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{\left(\sqrt{\rho^2 + z'^2}\right)^3} = \frac{\rho_L \, dz'}{4\pi\epsilon_0} \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{(\rho^2 + z'^2)^{3/2}}$$

But only E_ρ is present, so

$$dE_\rho = \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$\Rightarrow E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}} = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2 + z'^2)^{3/2}}$$

From the figure $\dfrac{z'}{\rho} = \cot\theta \Rightarrow z' = \rho\cot\theta$

$\Rightarrow dz' = \rho(-\operatorname{cosec}^2\theta)\,d\theta$

$$E_\rho = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{-\rho \operatorname{cosec}^2\theta \, d\theta}{(\rho^2 + \rho^2 \cot^2\theta)^{3/2}}$$

$$= \frac{\rho_L \rho (-\rho)}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{\operatorname{cosec}^2\theta \, d\theta}{[\rho^2(1+\cot^2\theta)]^{3/2}}$$

$$= \frac{-\rho_L \rho^2}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{\operatorname{cosec}^2\theta \, d\theta}{(\rho^2 \operatorname{cosec}^2\theta)^{3/2}}$$

$$= \frac{-\rho^2 \rho_L}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{\operatorname{cosec}^2\theta \, d\theta}{\rho^3 \operatorname{cosec}^3\theta}$$

$$= \frac{-\rho_L}{4\pi\epsilon_0 \rho} \int_{-\infty}^{\infty} \frac{d\theta}{\operatorname{cosec}\theta} = \frac{-\rho_L}{4\pi\epsilon_0 \rho} \int_{-\infty}^{\infty} \sin\theta \, d\theta$$

$$= \frac{-\rho_L}{4\pi\epsilon_0 \rho} \left[-\cos\theta\right]_{z'=-\infty}^{z'=\infty}$$

$$E_\rho = \frac{\rho_L}{4\pi\epsilon_0 \rho} \left. \frac{z'}{\sqrt{\rho^2 + z'^2}} \right|_{z'=-\infty}^{z'=+\infty}$$

$z' = \rho \cot\theta = \rho \frac{\cos\theta}{\sin\theta}$

As $z' \to \infty$, $\theta \to 0$

$z' \to -\infty$, $\theta \to \pi$

$$\boxed{E_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho}}$$

2nd Method Using θ as variable of integration

$$dE_\rho = dE \sin\theta = \left(\frac{\rho_L \, dz'}{4\pi\epsilon_0 R^2}\right) \sin\theta$$

$$= \frac{\rho_L (-\rho \operatorname{cosec}^2\theta \, d\theta) \sin\theta}{4\pi\epsilon_0 R^2}$$

$$= \frac{\rho_L (-\rho \operatorname{cosec}^2\theta \, d\theta) \sin\theta}{4\pi\epsilon_0 \cdot \rho^2 \operatorname{cosec}^2\theta}$$

$\frac{R}{\rho} = \operatorname{cosec}\theta$

$\Rightarrow R = \rho \operatorname{cosec}\theta$

$$dE_\rho = \frac{-\rho_L \sin\theta \, d\theta}{4\pi\epsilon_0 \rho}$$

$$\Rightarrow E_\rho = \frac{-\rho_L}{4\pi\epsilon_0 \rho} \int_{\pi}^{0} \sin\theta \, d\theta = \frac{\rho_L}{4\pi\epsilon_0 \rho} \cos\theta \Big|_{\pi}^{0} = \frac{\rho_L}{2\pi\epsilon_0 \rho}$$

3rd Method Starting from

$$\vec{E} = \int_{vol} \frac{\rho_v \, dv' \, (\vec{r}-\vec{r}\,')}{4\pi\epsilon_0 \, |\vec{r}-\vec{r}\,'|^3}$$

let $\rho_v \, dv' = \rho_L \, dz'$

$P(\rho, \phi, z)$

$\vec{r} = \rho \hat{a}_\rho + \cancel{\phi \hat{a}_\phi} + z \hat{a}_z$ — E_ϕ is ignored as it is zero

$\vec{r}\,' = z' \hat{a}_z$

$\vec{R} = \vec{r} - \vec{r}\,' = \rho \hat{a}_\rho + (z-z')\hat{a}_z$

$|\vec{R}| = \sqrt{\rho^2 + (z-z')^2}$

$$\hat{a}_R = \frac{\rho \hat{a}_\rho + (z-z')\hat{a}_z}{\sqrt{\rho^2+(z-z')^2}}$$

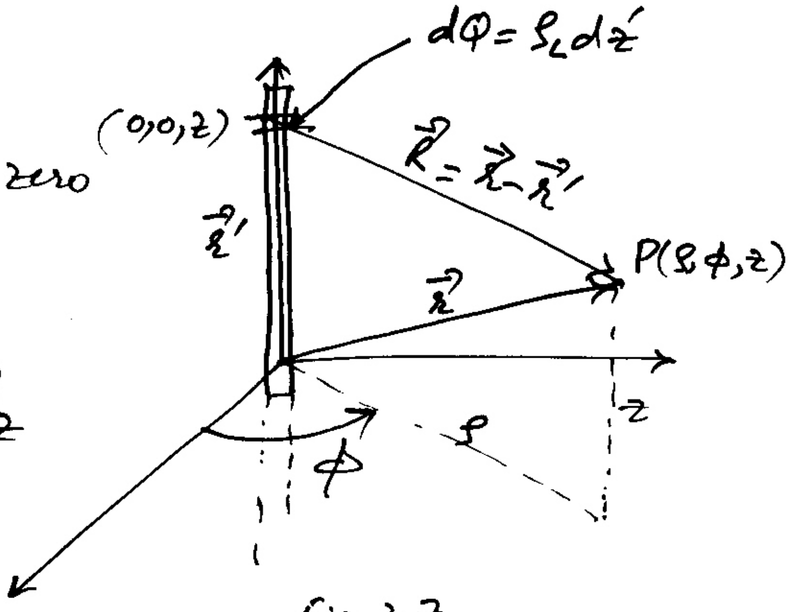


Fig 2.7

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_L \, dz' \left[\rho \hat{a}_\rho + (z-z')\hat{a}_z\right]}{4\pi\epsilon_0 \left[\rho^2 + (z-z')^2\right]^{3/2}}$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0} \left\{ \int_{-\infty}^{\infty} \frac{\rho \, dz' \, \hat{a}_\rho}{\left[\rho^2+(z-z')^2\right]^{3/2}} + \int_{-\infty}^{\infty} \frac{(z-z')\,dz' \, \hat{a}_z}{\left[\rho^2+(z-z')^2\right]^{3/2}} \right\}$$

→ Here $\hat{a}_\rho$ does not vary with z', so it is taken as constant.

→ Also $\hat{a}_z$ is always constant (does not vary with z'), so it is also taken as constant

$$\Rightarrow \vec{E} = \frac{\rho_L}{4\pi\epsilon_0} \left\{ \hat{a}_\rho \int_{-\infty}^{\infty} \frac{\rho \, dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}} + \hat{a}_z \int_{-\infty}^{\infty} \frac{(z-z')\,dz'}{\left[\rho^2+(z-z')^2\right]^{3/2}} \right\}$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left\{ a_\rho \cdot \rho \frac{1}{\rho^2} \left. \frac{-(z-z')}{\sqrt{\rho^2+(z-z')^2}} \right|_{-\infty}^{\infty} + \hat{a}_z \left. \frac{1}{\sqrt{\rho^2+(z-z')^2}} \right|_{-\infty}^{\infty} \right\}$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0}\left[\hat{a}_\rho \frac{2}{\rho} + \hat{a}_z(0)\right] = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho$$

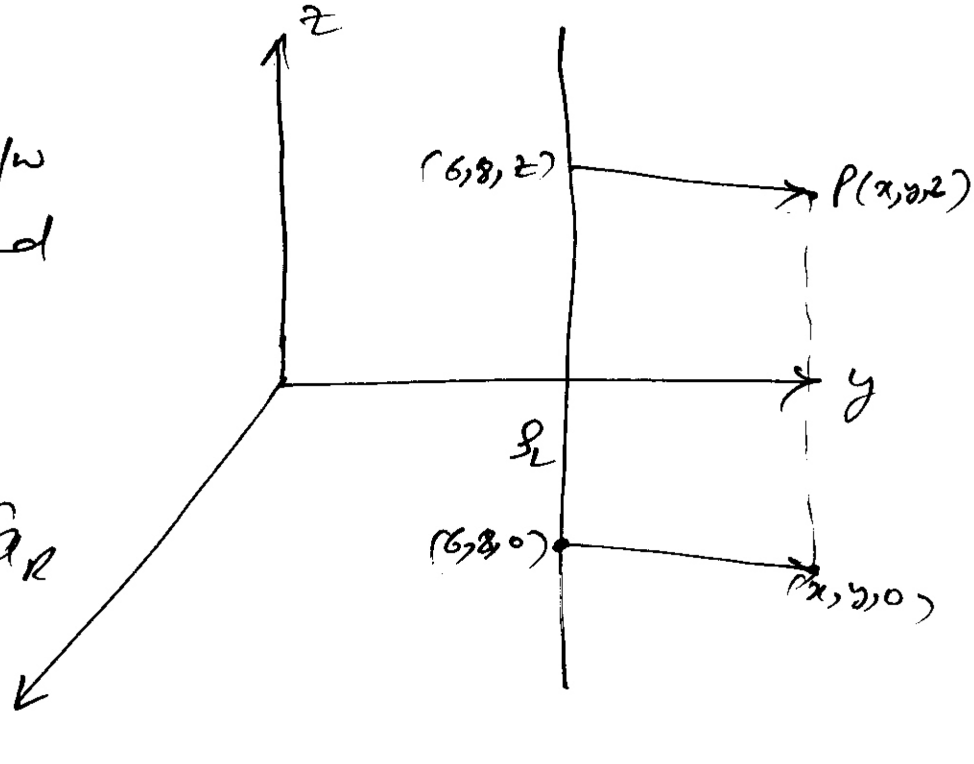
→ $\vec{E} \propto \frac{1}{\rho}$ in case of line of charge

→ $\vec{E} \propto \frac{1}{\rho^2}$ in case of point charge.

Line Of Charge ∥ to z-axis :

$\vec{E} = ?$ at $P(x,y,z)$

ρ = Radial distance b/w the line charge and point $P = R_0 = ?$



$$R = \sqrt{(x-6)^2+(y-8)^2}$$

$$\Rightarrow \vec{E} = \frac{\rho_L}{2\pi\epsilon_0\sqrt{(x-6)^2+(y-8)^2}}\hat{a}_R$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$$\hat{a}_R = \frac{(x-6)\hat{a}_x + (y-8)\hat{a}_y}{\sqrt{(x-6)^2+(y-8)^2}}$$

$$\Rightarrow \vec{E} = \frac{\rho_L}{2\pi\epsilon_0}\cdot\frac{(x-6)\hat{a}_x + (y-8)\hat{a}_y}{(x-6)^2+(y-8)^2}$$

Field Of A Sheet Of Charge

→ surface charge density ρ_s C/m²

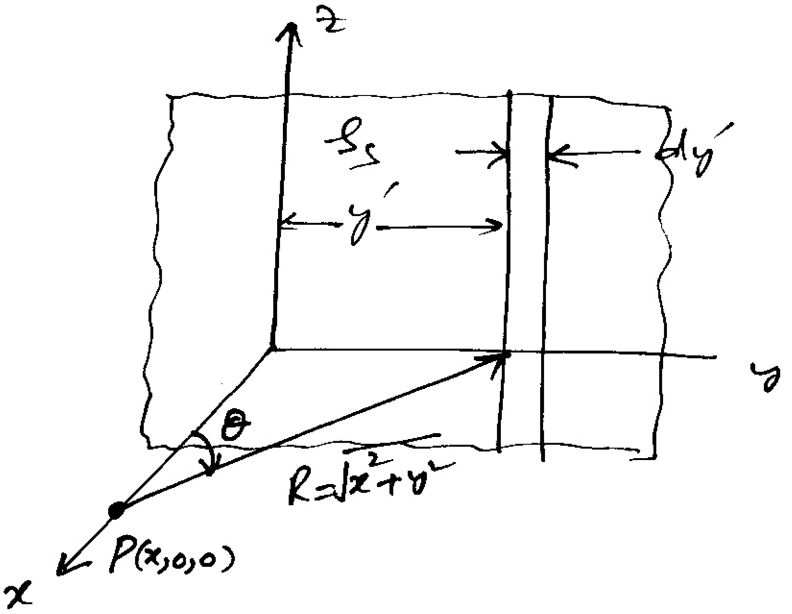
→ sheet of charge in the yz-plane

→ Field does not vary with y or z.

→ We divide infinite sheet into differential width strips (infinite line charges)

$\rho_L = \rho_s \, dy'$

$R = \sqrt{x^2 + y'^2}$



$$dE_x = \frac{\rho_s \, dy'}{2\pi\epsilon_0 \sqrt{x^2+y'^2}} \cos\theta$$

$$= \frac{\rho_s}{2\pi\epsilon_0} \cdot \frac{dy'}{\sqrt{x^2+y'^2}} \cdot \frac{x}{\sqrt{x^2+y'^2}}$$

$$dE_x = \frac{\rho_s}{2\pi\epsilon_0} \cdot \frac{x\, dy'}{(x^2+y'^2)}$$

$$\Rightarrow E_x = \frac{\rho_s}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{x\, dy'}{(x^2+y'^2)} = \frac{\rho_s}{2\pi\epsilon_0} \tan^{-1}\frac{y'}{x} \Big|_{-\infty}^{\infty}$$

$$\boxed{E_x = \frac{\rho_s}{2\epsilon_0}}$$

→ If P is chosen on the -ve x-axis, then

$$E_x = -\frac{\rho_s}{2\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_N$$

, Where $\hat{a}_N$ is outward normal to the sheet.

⇒ 2nd - Sheet Of Charge at $x = a$

★ In the region $x > a$,

$$E_+ = \frac{\rho_s}{2\epsilon_0} \hat{a}_x, \quad E_- = -\frac{\rho_s}{2\epsilon_0} \hat{a}_x, \quad E = E_+ + E_- = 0$$

★ For $x < 0$,

$$E_+ = -\frac{\rho_s}{2\epsilon_0} \hat{a}_x, \quad E_- = \frac{\rho_s}{2\epsilon_0} \hat{a}_x, \quad E = E_+ + E_- = 0$$

★ For $0 < x < a$,

$$\vec{E}_+ = \frac{\rho_s}{2\epsilon_0}\hat{a}_x, \quad \vec{E}_- = \frac{\rho_s}{2\epsilon_0}\hat{a}_x$$

$$\Rightarrow \vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho_s}{\epsilon_0}\hat{a}_x \quad \text{(doubled)}$$

Streamlines & Sketches Of Fields :

→ Streamlines or flux lines or direction lines sketch the magnitude & direction of $\vec{E}$.

→ $\vec{E}$ direction is found by taking tangent at any point on the streamline.

→ $\vec{E}$ strength is found by noting the spacing b/w the streamlines.

(OR) $\vec{E}$ strength is inversely proportional to the spacing b/w the stream lines.

→ For xy-plane, i.e, $E_z = 0$

$$\frac{E_y}{E_x} = \frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

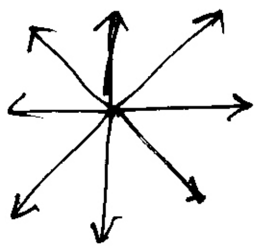
$$\Rightarrow \ln y = \ln x + \ln c$$

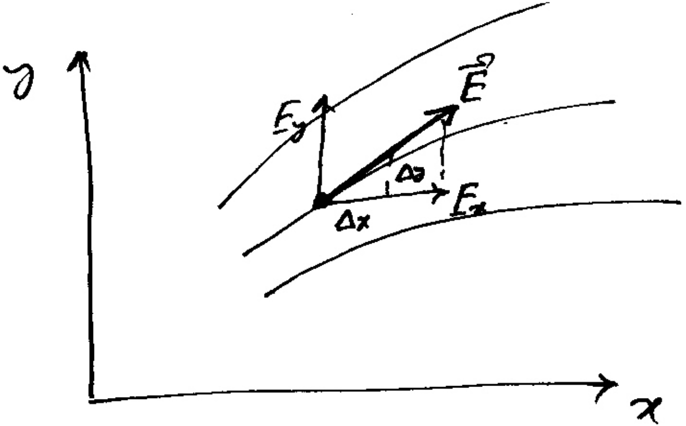
$$\Rightarrow \boxed{y = cx} \longrightarrow \text{Equation of streamlines}$$

→ Equation of one particular streamline passing through $P(-2, 7, 10)$ is

$$y = cx \Rightarrow 7 = c(-2) \Rightarrow c = -3.5$$

$$\Rightarrow \boxed{y = -3.5x}$$





y
x
$\vec{E}$
E_y
E_x
Δy
Δx